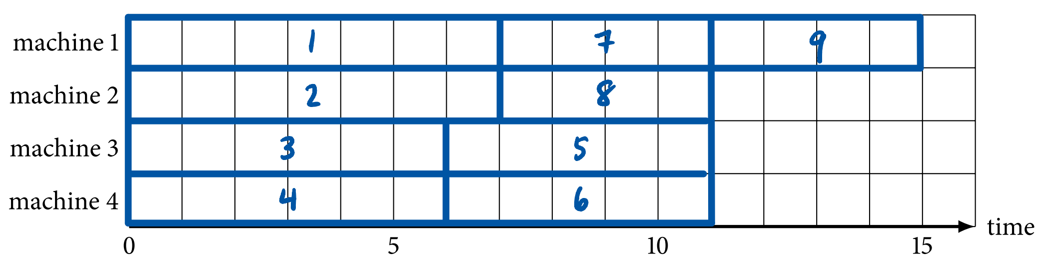
Lesson 9. Machine Scheduling

Example 1. The Markov Micromanufacturing Company has 9 production jobs it needs to process in the next 24 hours. The company has 4 identical machines that run in parallel. Each of these 9 jobs must be run on one of these machines **nonpreemptively**: that is, once a job is started on a machine, it must stay on that machine until it is completed. The processing times of these jobs are given below:

job	1	2	3	4	5	6	7	8	9
processing time (hours)	7	7	6	6	5	5	4	4	4

The company wants to minimize the **makespan**, or the completion time of the last job to finish processing.

- Let m be the number of machines — in this case, $m = 4$
- Suppose we schedule the jobs using the **longest processing time first (LPT)** rule:
    - First, schedule the m longest jobs on the m machines
    - Whenever a machine becomes free, put the longest unprocessed job on that machine
- Idea: LPT puts shorter jobs towards the end of the schedule, where they can be used to balance the loads on each machine
- For our problem, this yields a schedule that looks like this:

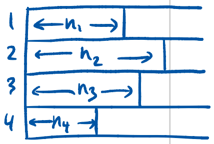


   - This kind of diagram is known as a **Gantt chart**

- Therefore, the makespan for the LPT schedule is $\boxed{15}$
- It turns out that the makespan of an LPT schedule is <u>always</u> at most $33.\overline{3}\%$ larger than the minimum makespan
- So... can we do better?
- Let's formulate this problem as a dynamic program
- Stages:

Stage t $\longleftrightarrow$ assigning job t to a machine $(t = 1, \dots, 9)$
$\longleftrightarrow$ end of decision-making process $(t = 10)$

- States in stage t (nodes):



load = amt. of processing assigned to a machine

stage t $\rightarrow$ t_{n_1, n_2, n_3, n_4}

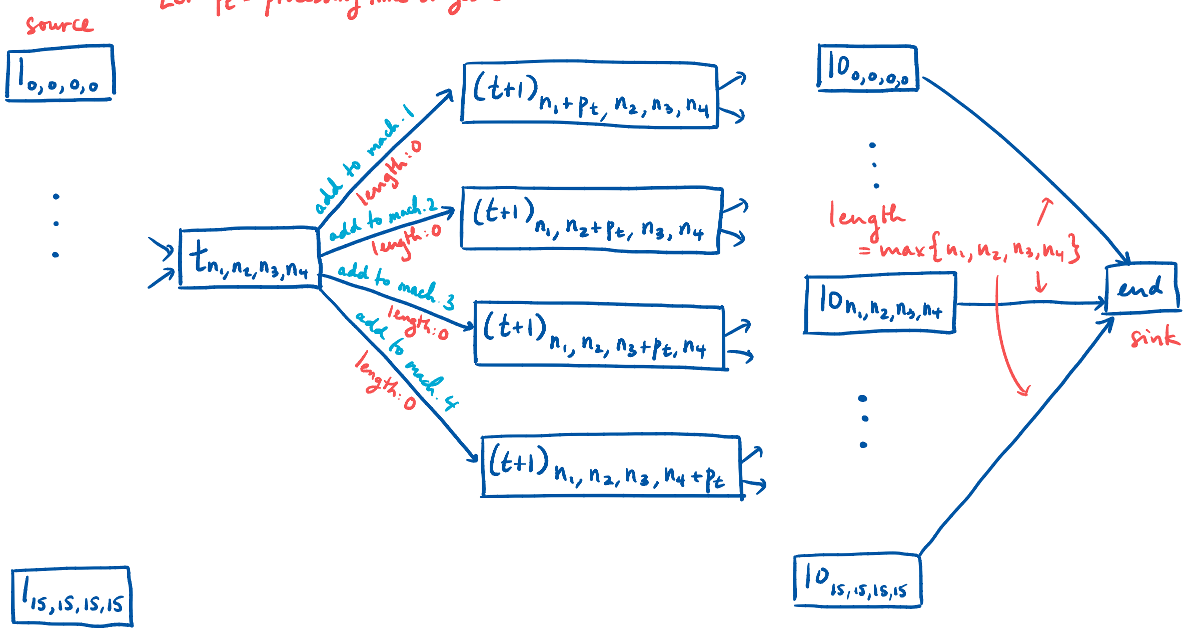
load on machine 1, load on machine 2, load on machine 3, load on machine 4

$n_1, n_2, n_3, n_4 = 0, 1, 2, \ldots, 15$

We know that the load on each machine ≤ 15 in an optimal solution b/c of LPT.

- Decisions, transitions, and rewards/costs at stage t (edges):

Let p_t = processing time of job t



- Shortest/longest path? shortest

- Minimum makespan $\longleftrightarrow$ length of a shortest path.

- Assignments of jobs to machines $\longleftrightarrow$

Examine edges in a shortest path:

e.g. $\left(t_{n_1, n_2, n_3, n_4} \, , \, (t+1)_{m_1, m_2, m_3, m_4}\right)$

If $n_i \neq m_i$, then assign job t to machine i

A Problems

See the accompanying Jupyter Notebook for this lesson.